

SPIRAL CURVES MADE SIMPLE

August 2009
Revised 2013

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□ HISTORY

- Spiral curves were originally designed for the Railroads to smooth the transition from a tangent line into simple curves. They helped to minimize the wear and tear on the tracks. Spiral curves were implemented at a later date on highways to provide a smooth transition from the tangent line into simple curves. The highway engineers later determined that most drivers will naturally make that spiral transition with the vehicle; therefore, spiral curves are only used on highways in special cases today.
- Because they were used in the past and in special cases today, we need to know how to calculate them.
- From the surveyor's perspective, the design of spiral curves has already been determined by the engineer and will be documented on existing R/W and As-built plans. All we have to do is use the information shown on these plans to fit the spiral curve within our surveyed alignment.

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□ REFERENCES

- There are many books available on spiral curves that can help you know and understand how the design process works. It can get complicated when you dive into the theory and design of spiral curves.
- My reference material includes the following books:
- Railroad Curves and Earthwork; by C. Frank Allen, S.B.
- Route Surveying and Design; by Carl F. Meyer & David W. Gibson
- Surveying Theory and Practice; by Davis, Foote & Kelly

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- **ADOT Roadway Guides for use in Office and Field 1986**
- This guide has all of the formulas and tables that you will need to work with spiral curves. The formulas, for the most part, are the same formulas used by the Railroad.
- The Railroads use the 10 Chord spiral method for layout and have tables setup to divide the spiral into 10 equal chords. Highway spirals can be laid out with the 10 Chord method but are generally staked out by centerline stationing depending on the needs in the field.
- For R/W calculations we only need to be concerned with the full spiral length.
- The tables that will be used the most are D-55.10 (Full Transition Spiral) and D-57.05 through D-57.95 (Transitional Spiral Tables).
- On rare occasions you may also need D-55.30 (Spiral Transition Compound Curves).

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□ **COURSE OBJECTIVE**

- This course is intended to introduce you to Spiral Curve calculations along centerline alignments.
- It is assumed that you already now how to calculate simple curves and generate coordinates from one point to another using a bearing and distance.
- Offsets to Spiral Curves and intersections of lines with Spiral Curves will not be discussed in this lesson. These types of calculations will be addressed in a future lesson.
- You can check your calculations using the online Spiral Calculator at:
- <http://www.cc4w.net/spiral/index.aspx>

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□ **EXAMPLE SPIRAL**

- Included are two example spiral curves from ADOT projects. The one that we will be calculating contains Equal Spirals for the Entrance and Exit on both sides of the main curve.
- The second example contains Unequal Spirals for the Entrance and Exit and a Transitional Spiral between two main curves with different radii. We will look at the process used to calculate this example but we will not be doing any calculations.
- The example spiral that we will be calculating is from the ADOT project along S.R. 64 as shown on sheet RS-17 of the Results of Survey.
- We will walk through each step to calculate this spiral.
- Note: My career started 30 plus years ago, before GPS and computers. I did all my calculations by hand and I teach my staff to do the calculations by hand so that they will have a thorough understanding of the mathematical process. I am a big advocate of technology and use it exclusively. I also have a passion for the art of surveying mathematics, therefore I feel that everyone should know how to do it manually. I feel that my staff has a better appreciation for technology by having done the calculations manually, at least once, before they rely on a computer to do it for them.

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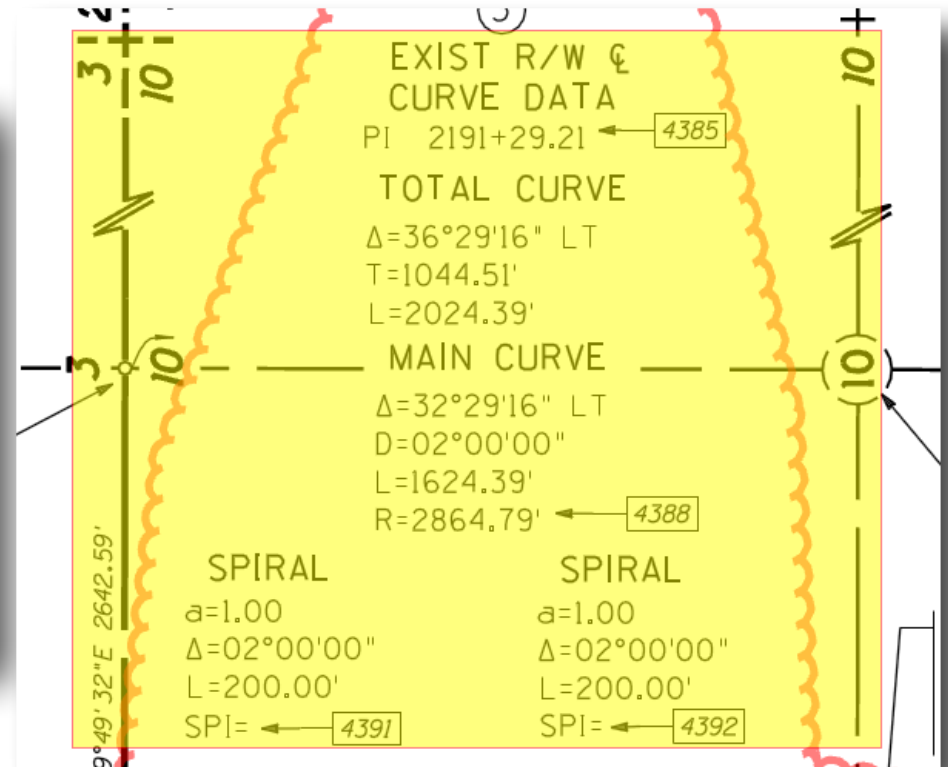
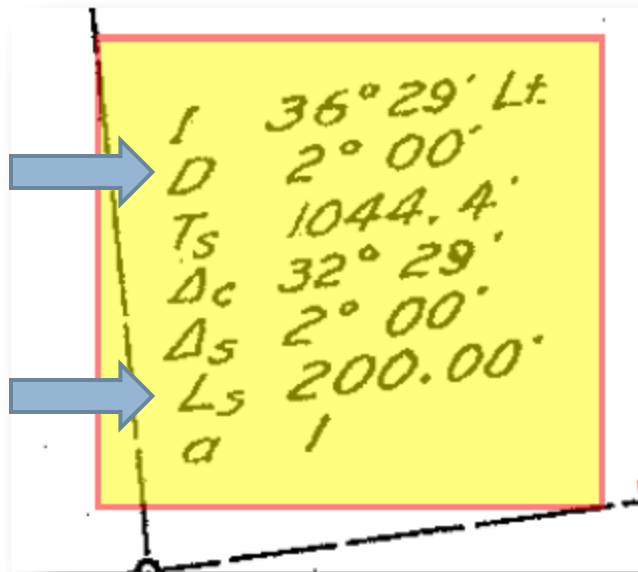
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- Look for the spiral curve and main curve information

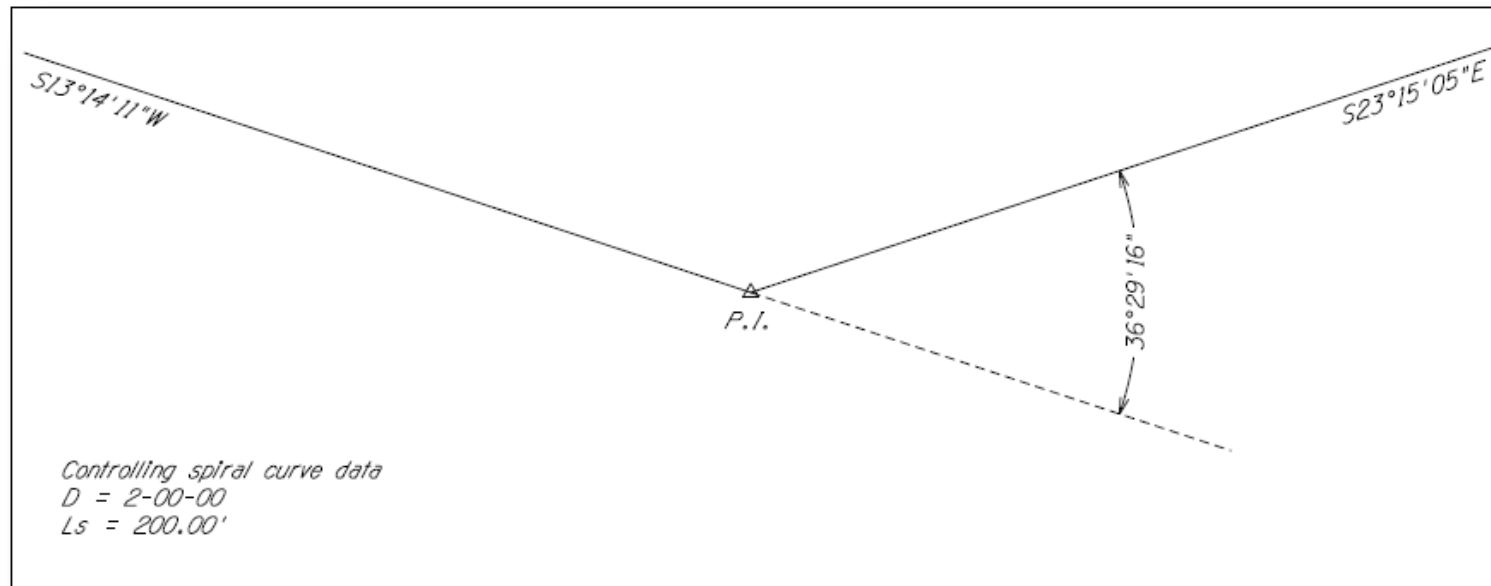


- The key information needed is the Degree of Curvature and the Spiral length.
- $D=2^{\circ}00'00''$ and $L_s=200.00'$

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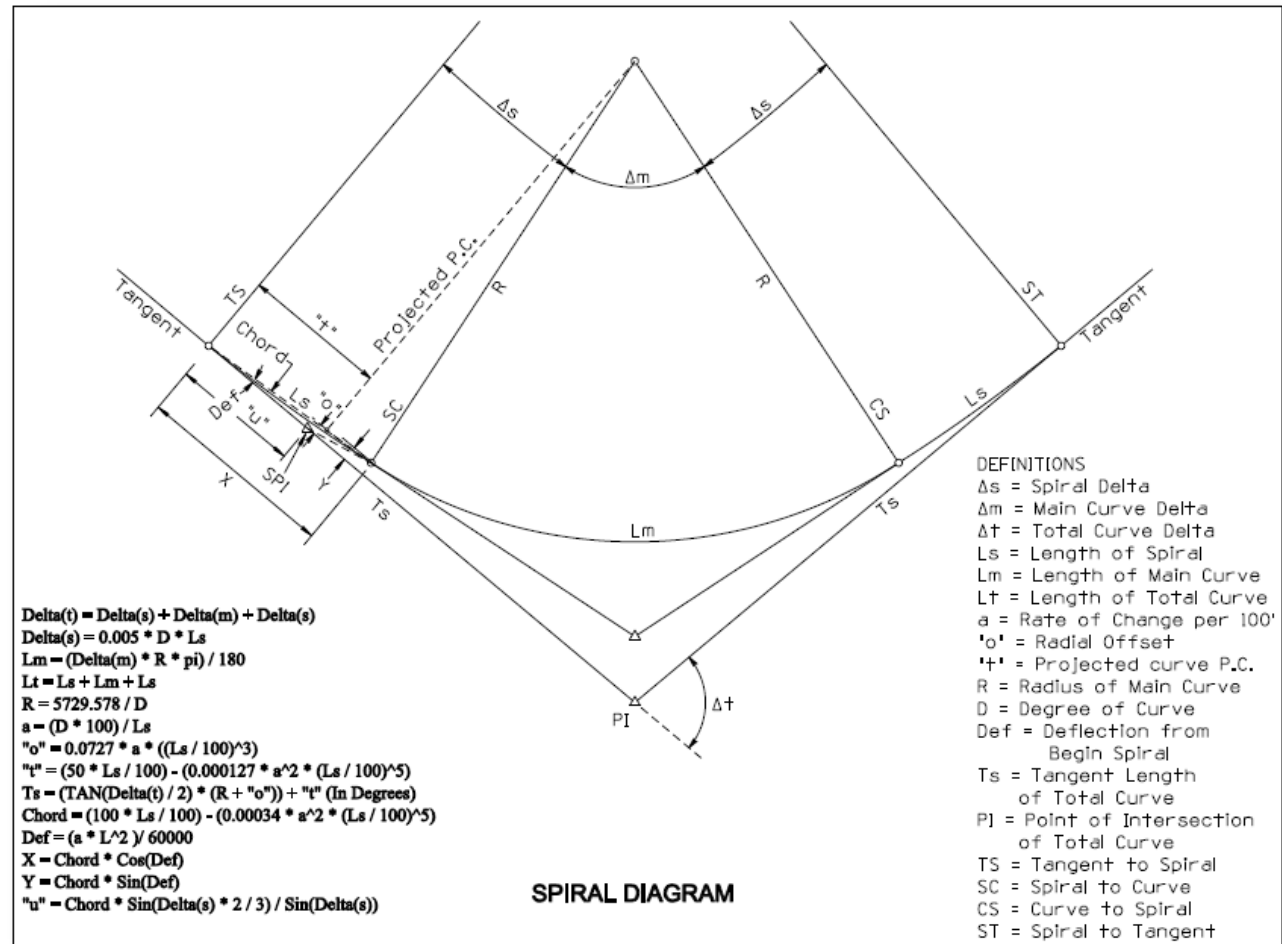
- **No. 2**
- Your tangent lines should be defined either by survey or record information. Sketch your tangent lines and Point of Intersection. Add the bearings for the tangent lines and calculate the deflection at the P.I. As shown below, the deflection is $36^{\circ}29'16''$.
- $13^{\circ}14'11'' + 23^{\circ}15'05'' = 36^{\circ}29'16''$



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- **No. 3**
- The following are spiral formulas that have been derived from several reference materials for spiral calculations that will be utilized for this lesson.



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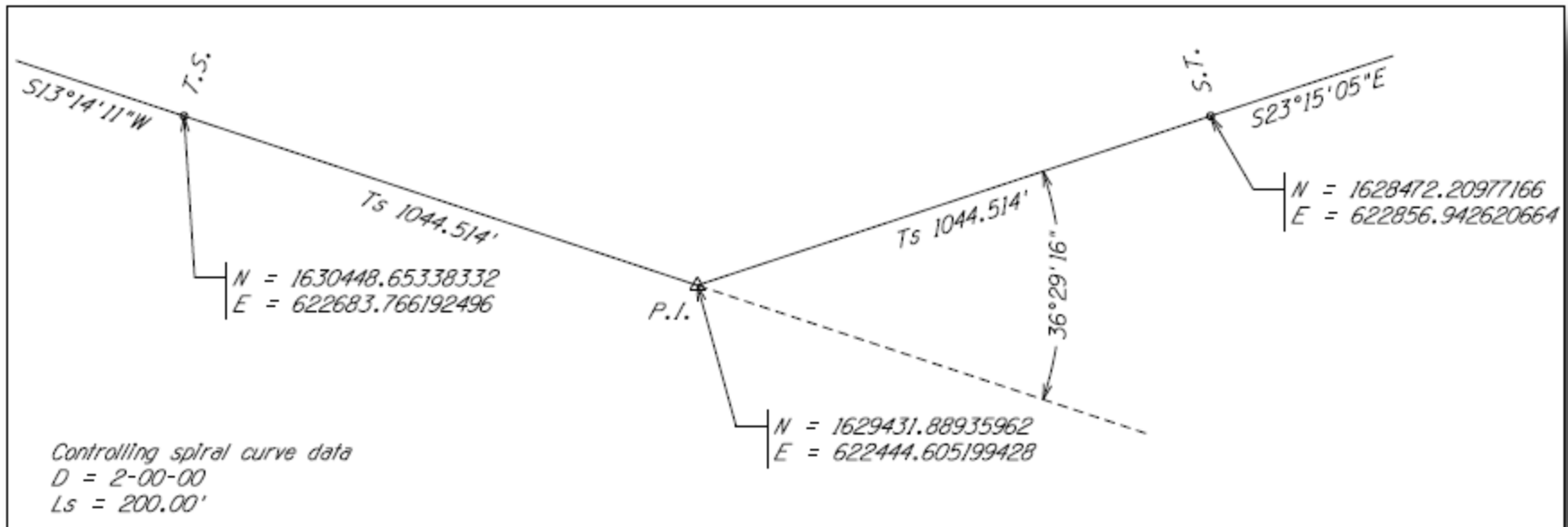
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- **No. 3 – continued**
- Next we will calculate the tangent distance (Ts) from the T.S. to the P.I.
- Use the following formulas to calculate Ts.
- $\Delta(t) = \underline{36-29-16(dms) \sim 36.48777777(ddd)}$
- $D = \underline{2-00-00(dms) \sim 2.0000(ddd)}$
- $L_s = \underline{200.00'}$
- $R = 5729.578 / D = 5729.578 / 2.0000 = \underline{2864.789'}$
- $\alpha = (D * 100) / L_s = (2.0000 * 100) / 200.00 = \underline{1.00}$ (Checks with record data)
- $“o” = 0.0727 * \alpha * ((L_s / 100)^3)$
 - $“o” = 0.0727 * 1 * ((200.00 / 100)^3) = \underline{0.5816}$
- $“t” = (50 * L_s / 100) - (0.000127 * \alpha^2 * (L_s / 100)^5)$
 - $“t” = (50 * 200.00 / 100) - (0.000127 * 1^2 * (200.00 / 100)^5) = \underline{99.9959}$
- $T_s = (\tan(\Delta(t) / 2) * (R + “o”)) + “t”$
 - $T_s = (\tan(36.48777777 / 2) * (2864.789 + 0.5816)) + 99.9959 = \underline{1044.515'}$

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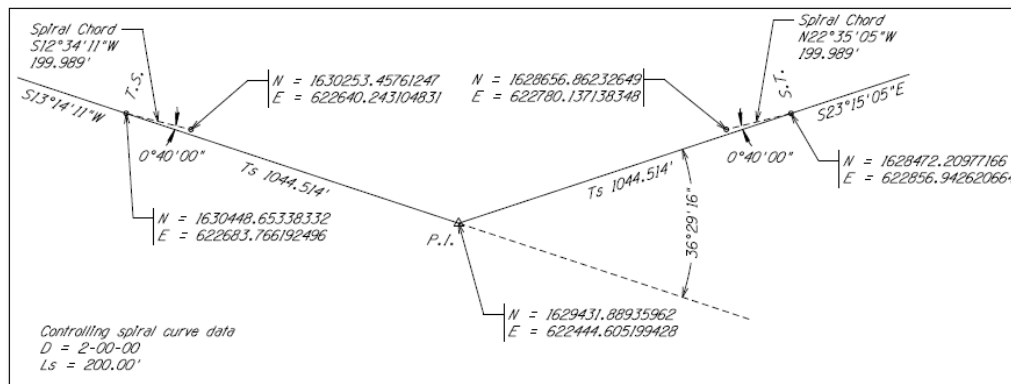
- **No. 3 – continued**
- Calculate the Northing and Easting for the Tangent to Spiral (T.S.) & Spiral to Tangent (S.T.)
- Use coordinate geometry to calculate the Latitude and Departure for each course and add them to the Northing and Easting of the Point of Intersection (P.I.) to get the Northing and Easting for the T.S. and S.T.



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- **No. 4**
- Calculate the spiral chord distance (Chord) and deflection angle (Def).
- $\text{Chord} = (100 * L_s / 100) - (0.00034 * \alpha^2 * (L_s / 100)^5)$
 - ▣ $\text{Chord} = (100 * 200.00 / 100) - (0.00034 * 1^2 * (200.00 / 100)^5) = \underline{199.989'}$
- $\text{Def} = (\alpha * L_s^2) / 60000 = (1 * 200.00^2) / 60000 = \underline{0.666666(ddd) \sim 0-40-00(dms)}$
- Calculate the chord bearing and Northing & Easting for the Spiral to Curve (S.C.) & Curve to Spiral (C.S.)
- Note: Substitute L_s for any length (L) along the spiral to calculate the sub-chord and Def angle to any point along the spiral from the T.S.

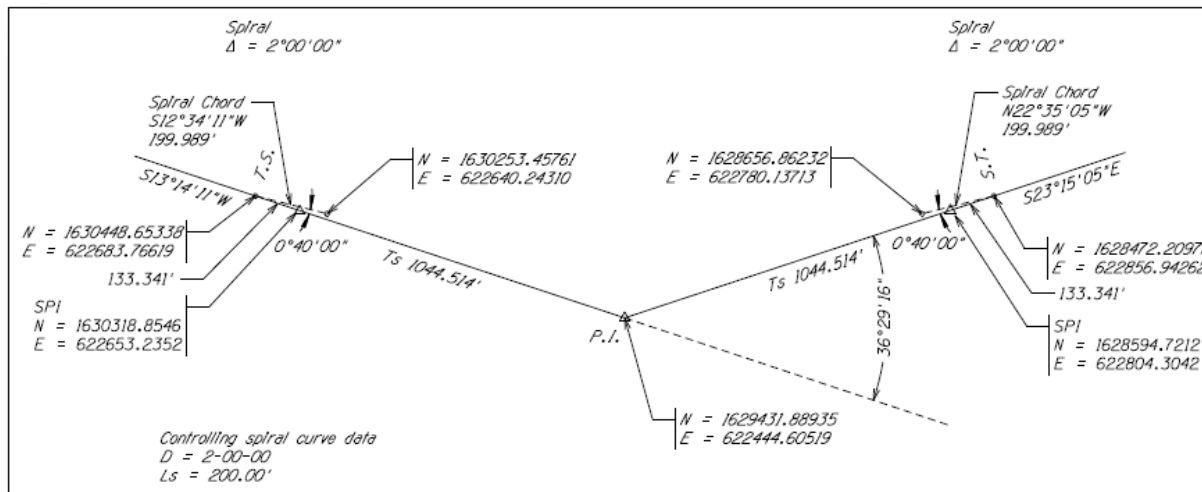


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- **No. 5**
- Calculate the spiral delta and tangent distance to the Spiral Point of Intersection (SPI).
- $\Delta(s) = 0.005 * D * L_s = 0.005 * 2.0000 * 200.00 = \underline{2.0000(ddd) \sim 2-00-00(dms)}$
- “u” = Chord * Sin(Delta(s) * 2 / 3) / Sin(Delta(s))
 - ▣ “u” = $199.989 * \sin(2.0000 * 2 / 3) / \sin(2.0000) = \underline{133.341'}$
- Calculate Northing and Easting of SPI.

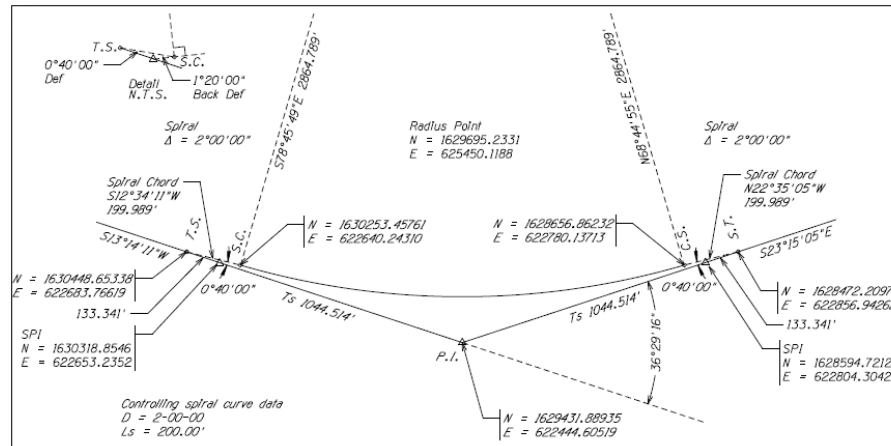


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- **No. 6**
- Calculate the radial line and radius point for the main curve.
- Calculate the back deflection from the S.C. to the SPI is as follows:
- $\Delta(s) - \text{Def} = \text{Back Def}$
 - $2.0000 - 0.666666 = \underline{1.333334(ddd) \sim 1-20-00(dms)}$
- Using the Back Def and chord bearing calculate the tangent bearing at the S.C. then perpendicular from the tangent line calculate the radial line. Do this for the C.S. as well. Calculate the Radius point from the S.C. and C.S. You should come up with the same coordinates. If not, then something is wrong. Recheck all of your calculations.



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- **No. 7**
- Using the radial bearings calculate the Main Curve Delta of 32-29-16(dms) ~ 32.487777(dms)
- Now add the Spiral Delta as follows:
- $\Delta(t) = \Delta(s) + \Delta(m) + \Delta(s)$
 - ▣ $\Delta(t) = 2-00-00 + 32-29-16 + 2-00-00 = \underline{36-29-16}$ this should equal the deflection in Step 2.
- Calculate the arc length for Main Curve
- $L_m = (\Delta(m) * R * \pi) / 180$
 - ▣ $L_m = (32.487777 * 2864.789 * 3.141592654) / 180 = \underline{1624.389'}$

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- **No. 8 – Almost done**
- The final step is to calculate the stationing.
- Starting with the T.S. calculate each stationing along the curve.
- T.S. 2180+84.70 + 200.00 (Ls) = S.C. 2182+84.70
- S.C. 2182+84.70 + 1624.39 (Lm) = C.S. 2199+09.09
- C.S. 2199+09.09 + 200.00 (Ls) = S.T. 2201+09.09

- P.I. Stationing = T.S. Stationing + Ts
- T.S. 2180+84.70 + 1044.51 (Ts) = P.I. 2191+29.21

- Be aware that you may have some slight differences in the coordinates, distances and stationing due to rounding errors.

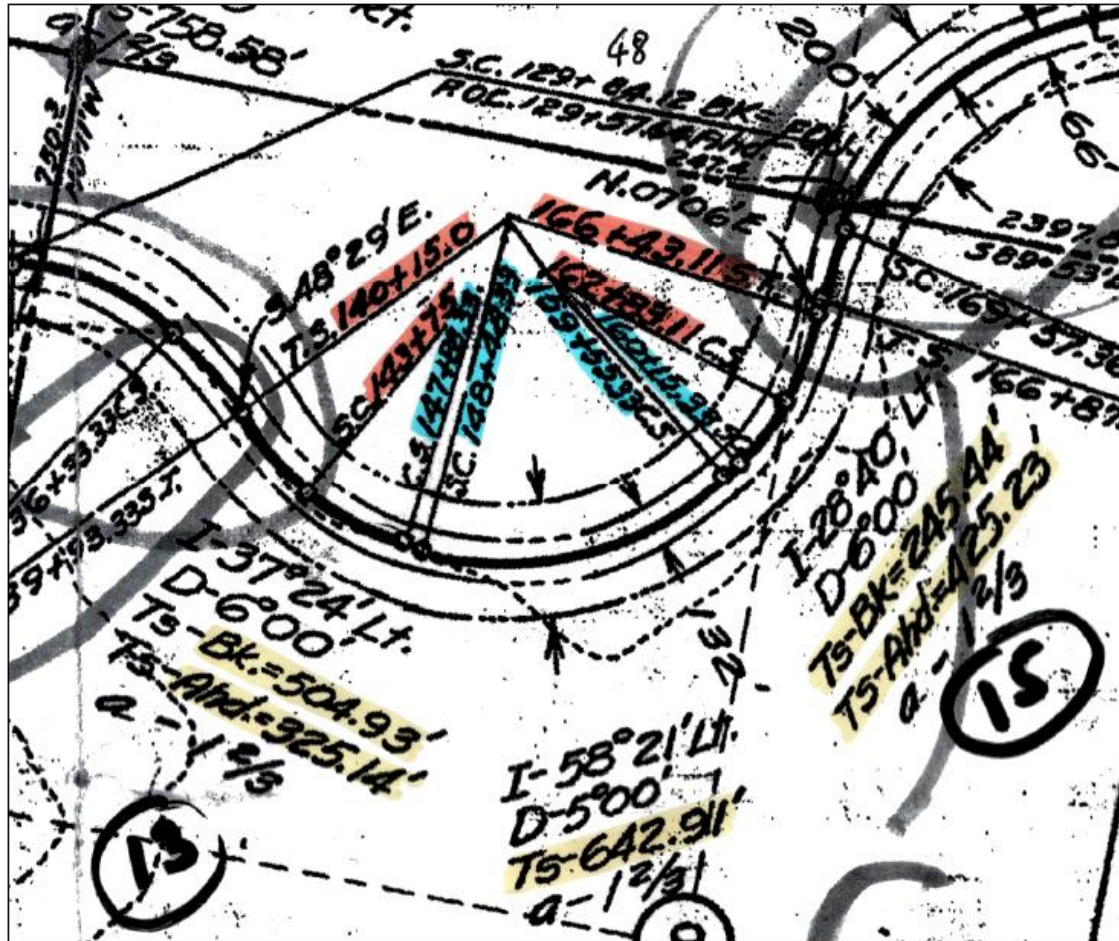
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- **Unequal Spiral information and Transitional Spirals**
- The following example is an ADOT project along U.S. 60 west of Globe AZ.
- The 10 miles section of highway is almost completely composed of spiral curves. There is one area that contained an entrance spiral, then a curve, then a transitional spiral, then a curve, then a transitional spiral, then a curve and finally an exit spiral.
- The next slide shows the record information for this segment.

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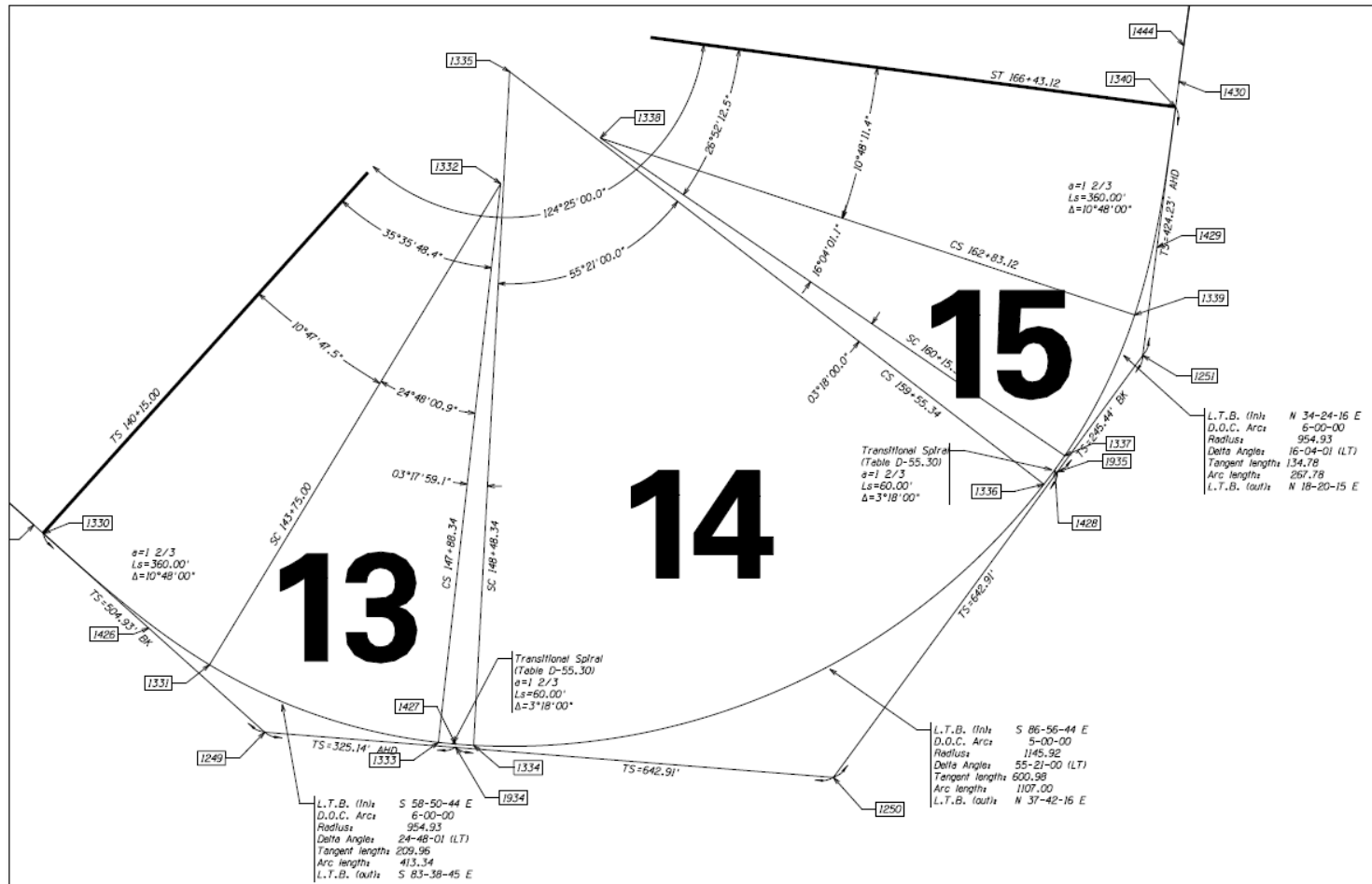
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MicroStation tools and manual calculations were used to solve for curves 13, 14 & 15.

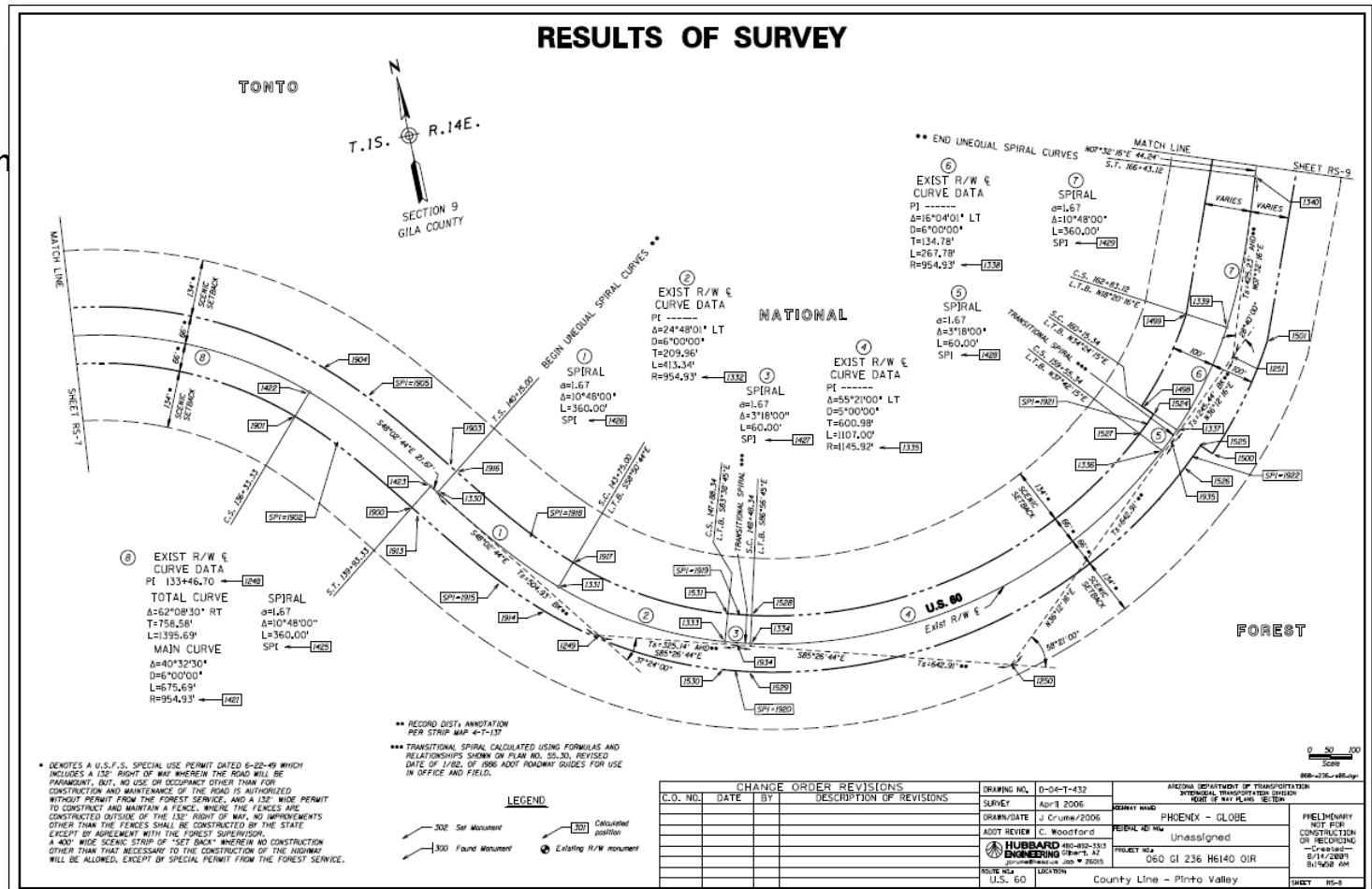
Table D-55.30 was used for the formulas needed to calculate the transitional spirals connecting the three main curves.



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Sheet RS-8 is how this multi-curve segment was shown on the Results of Survey



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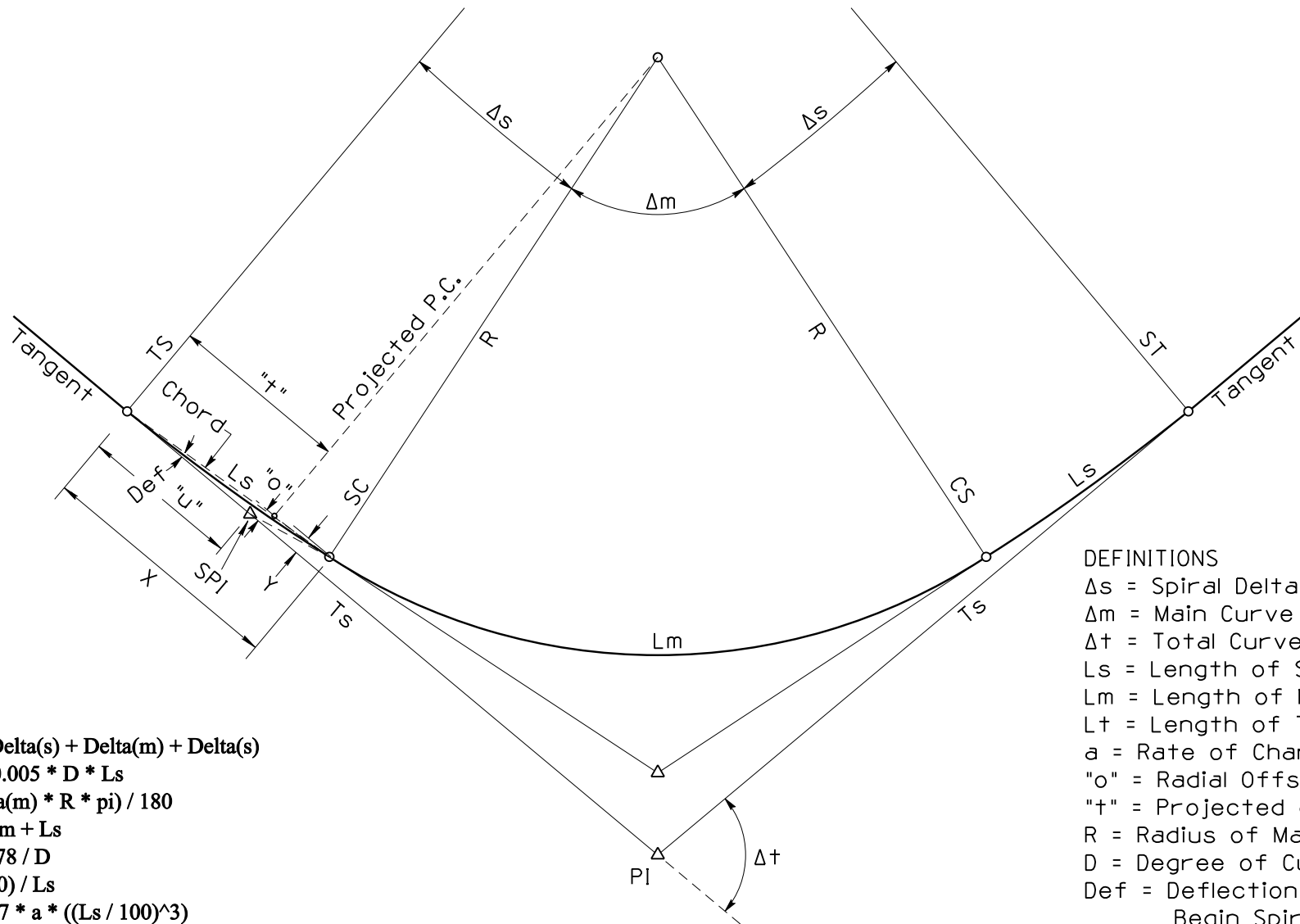
Calculating spiral curves does not have to be complicated. Once you understand the elements needed and methodically step through the process, you will obtain consistent results and might even have fun while doing it.

I hope that this presentation will debunk some of the myths that spiral curves are complicated and difficult to work with and will not make your hair turn gray.

You can contact me at jcrume@cc4w.net if you have any questions.

The Appendix contains full size PDF sheets that you can printout for your reference material.

APPENDIX



$$\Delta(t) = \Delta(s) + \Delta(m) + \Delta(s)$$

$$\Delta(s) = 0.005 * D * L_s$$

$$L_m = (\Delta(m) * R * \pi) / 180$$

$$L_t = L_s + L_m + L_s$$

$$R = 5729.578 / D$$

$$a = (D * 100) / L_s$$

$$"o" = 0.0727 * a * ((L_s / 100)^3)$$

$$"t" = (50 * L_s / 100) - (0.000127 * a^2 * (L_s / 100)^5)$$

$$T_s = (\tan(\Delta(t) / 2) * (R + "o")) + "t" \text{ (In Degrees)}$$

$$\text{Chord} = (100 * L_s / 100) - (0.00034 * a^2 * (L_s / 100)^5)$$

$$\text{Def} = (a * L^2) / 60000$$

$$X = \text{Chord} * \cos(\text{Def})$$

$$Y = \text{Chord} * \sin(\text{Def})$$

$$"u" = \text{Chord} * \sin(\Delta(s) * 2 / 3) / \sin(\Delta(s))$$

DEFINITIONS

Δs = Spiral Delta

Δm = Main Curve Delta

Δt = Total Curve Delta

L_s = Length of Spiral

L_m = Length of Main Curve

L_t = Length of Total Curve

a = Rate of Change per 100'

"o" = Radial Offset

"t" = Projected curve P.C.

R = Radius of Main Curve

D = Degree of Curve

Def = Deflection from

Begin Spiral

T_s = Tangent Length
of Total Curve

PI = Point of Intersection
of Total Curve

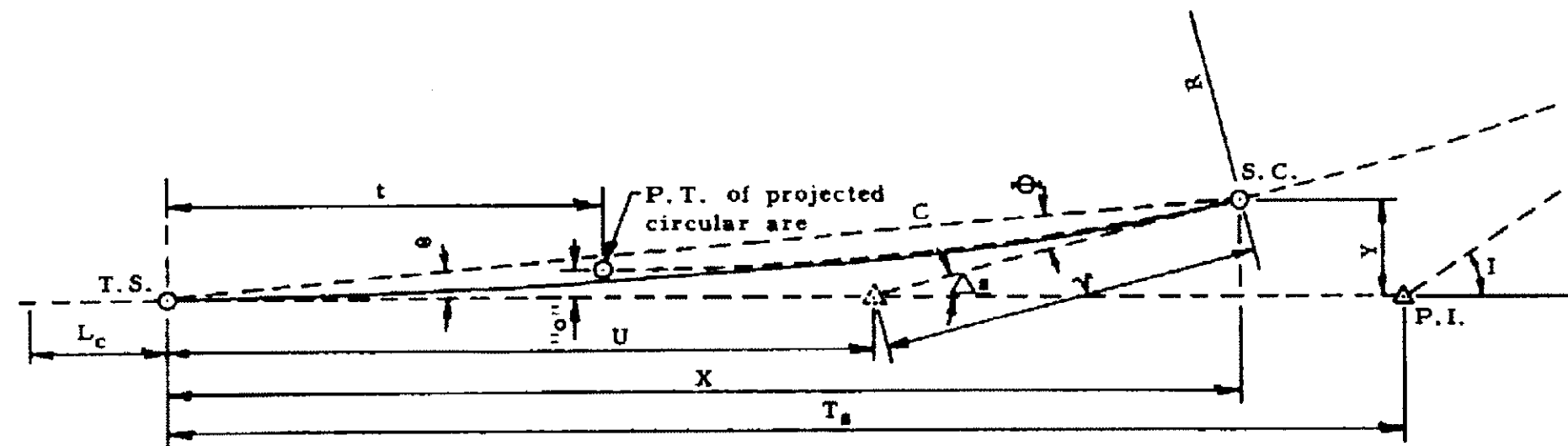
TS = Tangent to Spiral

SC = Spiral to Curve

CS = Curve to Spiral

ST = Spiral to Tangent

SPIRAL DIAGRAM



Determine values of L_s and a , for design speed and D , from Drwg. Nos. D-56.10 through D-56.30. These values may be checked by the following applicable formulae.

$L_s(\text{sta.}) = \text{Spiral distance from T.S. to S.C.} = D/a$
 $a(\text{degrees}) = \text{Rate of change in degree of curvature per 100' of spiral} = D/L_s$

$D(\text{degrees}) = \text{Degree of curvature of circular curve}$
 $= 5729.58/R$

$R(\text{ft.}) = \text{Radius of circular curve} = 5729.58/D$

"o"(ft.) = Radial offset $= 0.0727aL_s^3$

$t(\text{ft.}) = 50L_s - 0.000127a^2L_s^3$

$\Delta_s(\text{degrees}) = \text{Full spiral deviation angle} = \frac{1}{2}aL_s^2$
 $= \frac{1}{2}DL_s = \frac{1}{2}(D^2/a) = L_s/2(D)$

$\Theta(\text{degrees}) = \text{Full spiral deflection angle at T. S.}$
 $= (1/3\Delta_s)^{\circ} = 1/6aL_s^2 = 1/6DL_s = 1/6D^2/a^{\circ}$

$\phi(\text{degrees}) = \text{Full spiral deflection angle at S.C.}$
 $= \Delta_s - \Theta$

$C(\text{ft.}) = 100 L_s - 0.00034a^2L_s^3$

$V(\text{ft.}) = C \sin \Theta / \sin \Delta_s$

$U(\text{ft.}) = C \sin \phi / \sin \Delta_s$

$X(\text{ft.}) = C \cos \Theta$

$Y(\text{ft.}) = C \sin \Theta$

$T_s(\text{ft.}) = [(\tan \frac{1}{2}I)(R + "o")] + t$

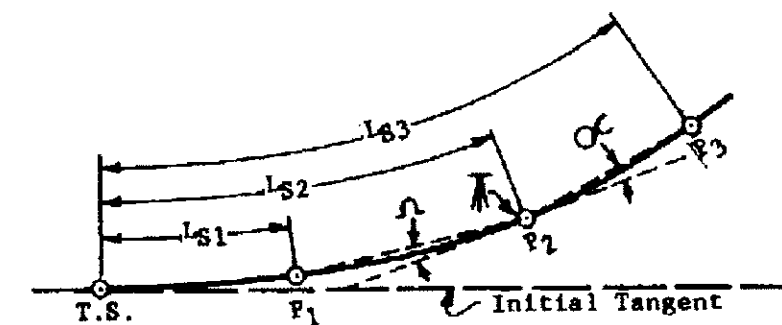
SPIRAL FORMULAE

Reduce Θ formulae values by C_n according to the following table:

Δ_s	C_n	Δ_s	C_n	Δ_s	C_n
16°	0.2'	31°	1.6'	46°	5.1'
17	0.3	32	1.7	47	5.5
18	0.3	33	1.9	48	5.8
19	0.4	34	2.1	49	6.2
20	0.4	35	2.3	50	6.6
21	0.5	36	2.5	51	7.0
22	0.6	37	2.7	52	7.4
23	0.6	38	2.9	53	7.9
24	0.7	39	3.1	54	8.3
25	0.8	40	3.4	55	8.8
26	0.9	41	3.6	56	9.3
27	1.0	42	3.9	57	9.8
28	1.2	43	4.2	58	10.3
29	1.3	44	4.5	59	10.8
30	1.4	45	4.8	60	11.4

Θ VALUES OF C_n IN Θ DETERMINATION FORMULAE

(C_n is negligible and may be ignored for Δ_s values less than 16°.)

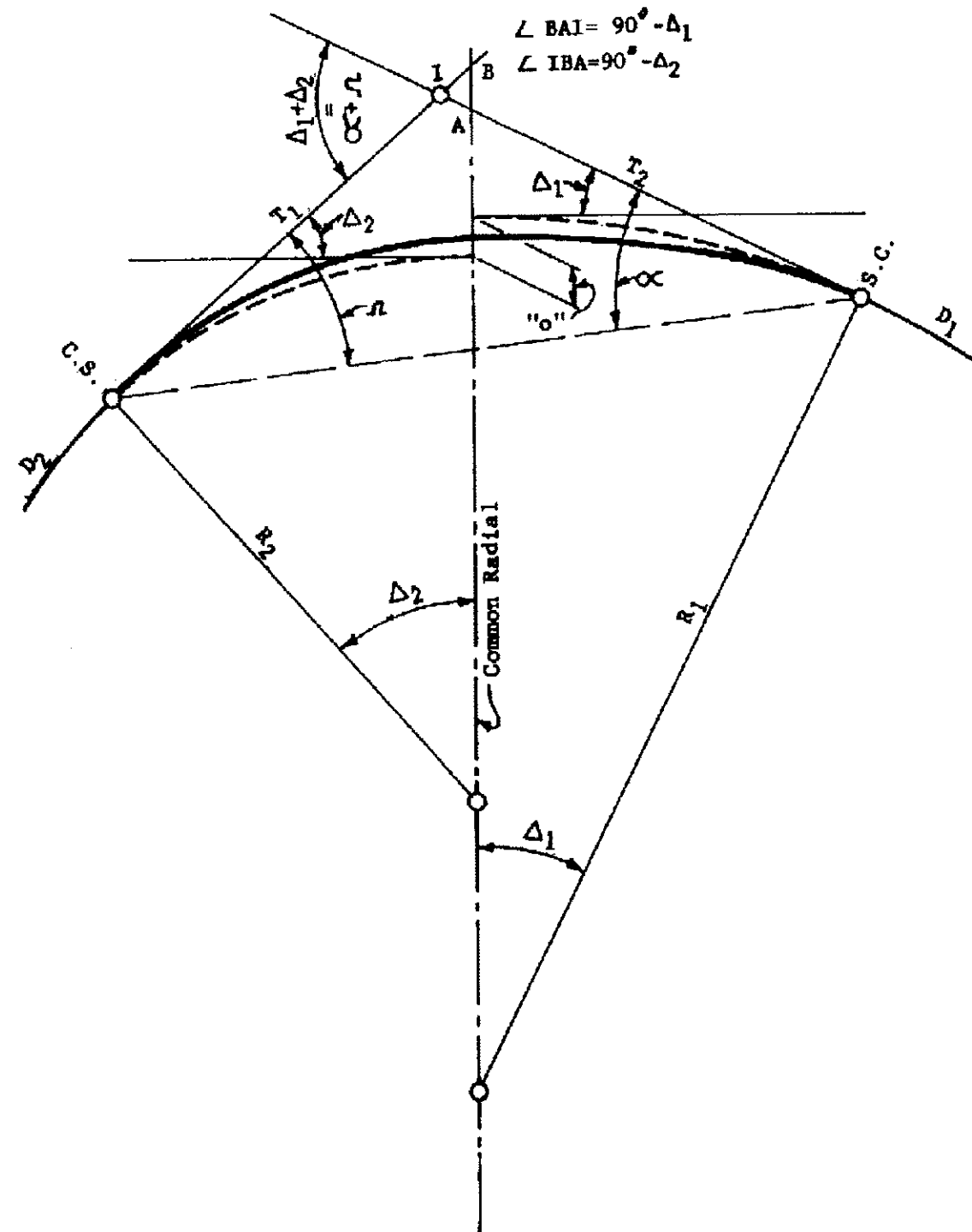


$$\alpha = \frac{1}{2}aL_{s2}(L_{s3} - L_{s2}) + 1/6a(L_{s3} - L_{s2})^2$$

$$\alpha = \frac{1}{2}aL_{s2}(L_{s2} - L_{s1}) - 1/6a(L_{s2} - L_{s1})^2$$

DEFLECTION ANGLE FORMULAE FOR SET-UP AT POINT ON SPIRAL

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APPROVED FOR DISTRIBUTION <i>[Signature]</i>	FULL TRANSITION SPIRAL	PLAN NO. D-55.10



Intermediate Spiral Transition is basically the same as Partial Transition Spiral illustrated by Drwg. No D-55.20.

Select L_s and a from Drwg. No. D-56.10. through Drwg. No. D-56.30 for design speed and $D=D_1-D_2$. These values are applied throughout the following formulae.

$$L_s(\text{sta.}) = (D_2 - D_1) / a$$

$$a(\text{degrees}) = (D_2 - D_1) L_s$$

$$D_p(\text{sta.}) = \text{Degree of curvature at any point on spiral.}$$

$$= D_2 - (a)(\text{distance in sta. from C.S. to point}).$$

$$= D_1 + (a)(\text{distance in sta. from S.C. to point}).$$

$$"o"(\text{ft.}) = 0.0727(D_2 - D_1) \left(\frac{D_2 - D_1}{a} \right)^2$$

$$\alpha(\text{degrees}) = 1/2 D_2 \left(\frac{D_2 - D_1}{a} \right) - 1/6 a \left(\frac{D_2 - D_1}{a} \right)^2$$

$$\alpha(\text{degrees}) = 1/2 D_1 \left(\frac{D_2 - D_1}{a} \right) + 1/6 a \left(\frac{D_2 - D_1}{a} \right)^2$$

To calculate deflections and spiral distance to any point on spiral, substitute D_p for D_1 or D_2

$$\Delta_1 = (\text{degrees}) = D_1(L_s/2)$$

$$\Delta_2 = (\text{degrees}) = D_2(L_s/2)$$

$$AB = R_2 \text{ in feet (exsec. } \Delta_2) - R_1 \text{ in feet (exsec. } \Delta_1) - "o"$$

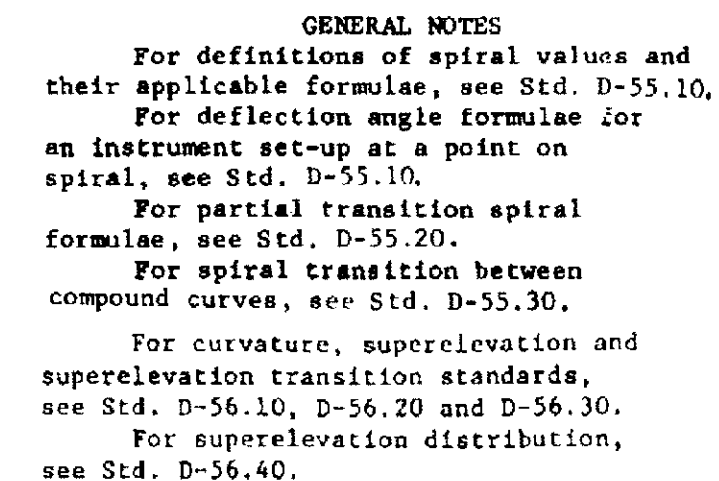
$$AI(\text{ft.}) = AB \left[\frac{\cos \Delta_2}{\sin(\Delta_1 + \Delta_2)} \right]$$

$$BI(\text{ft.}) = AB \left[\frac{\cos \Delta_1}{\sin(\Delta_1 + \Delta_2)} \right]$$

$$T_1(\text{ft.}) = R_1 \text{ in feet } (\tan \Delta_1) + AI$$

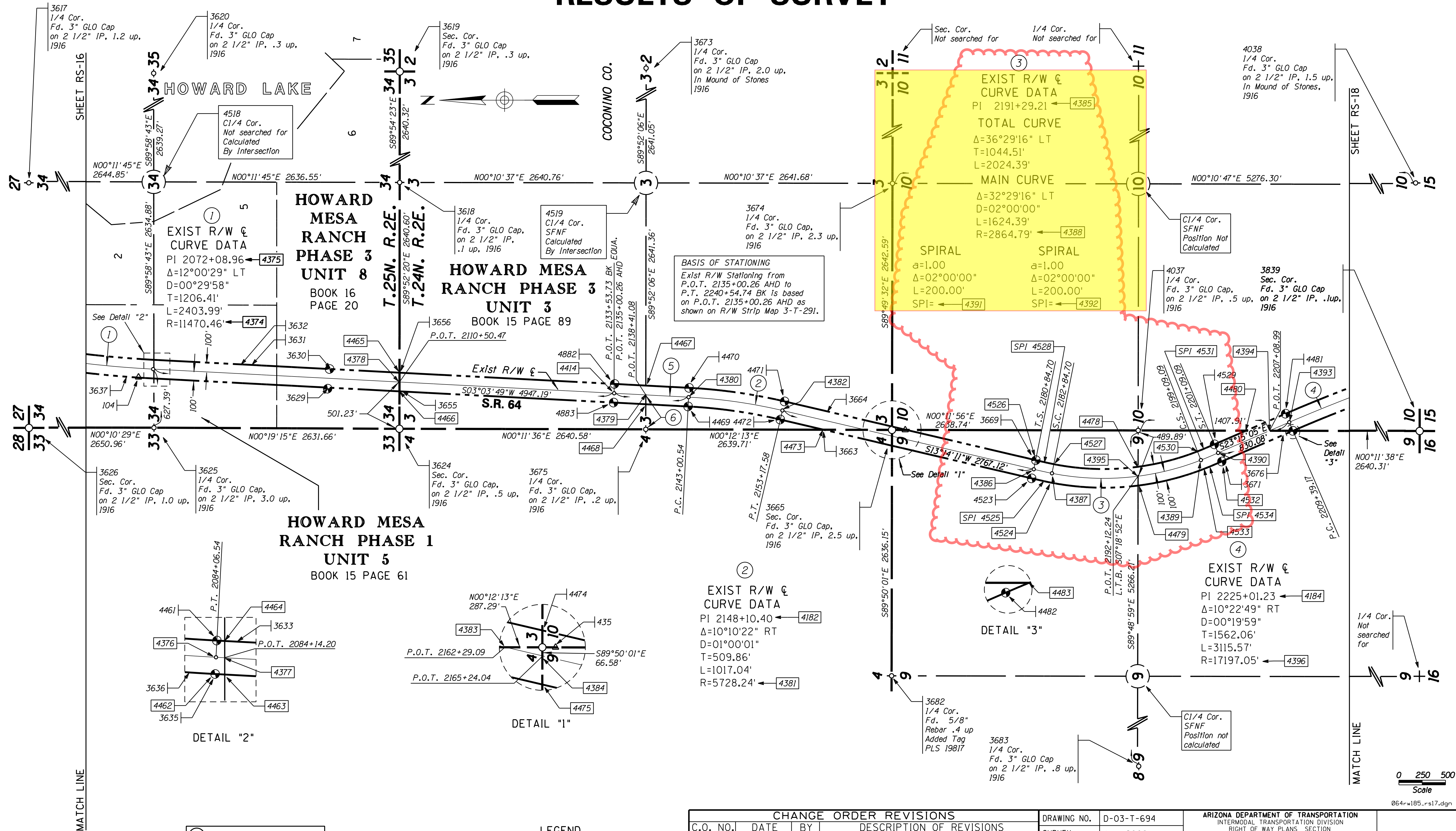
$$T_2(\text{ft.}) = R_2 \text{ in feet } (\tan \Delta_2) - BI$$


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APPROVED FOR DISTRIBUTION <i>[Signature]</i>	SPIRAL TRANSITION COMPOUND CURVES	PLAN NO. D-55.30

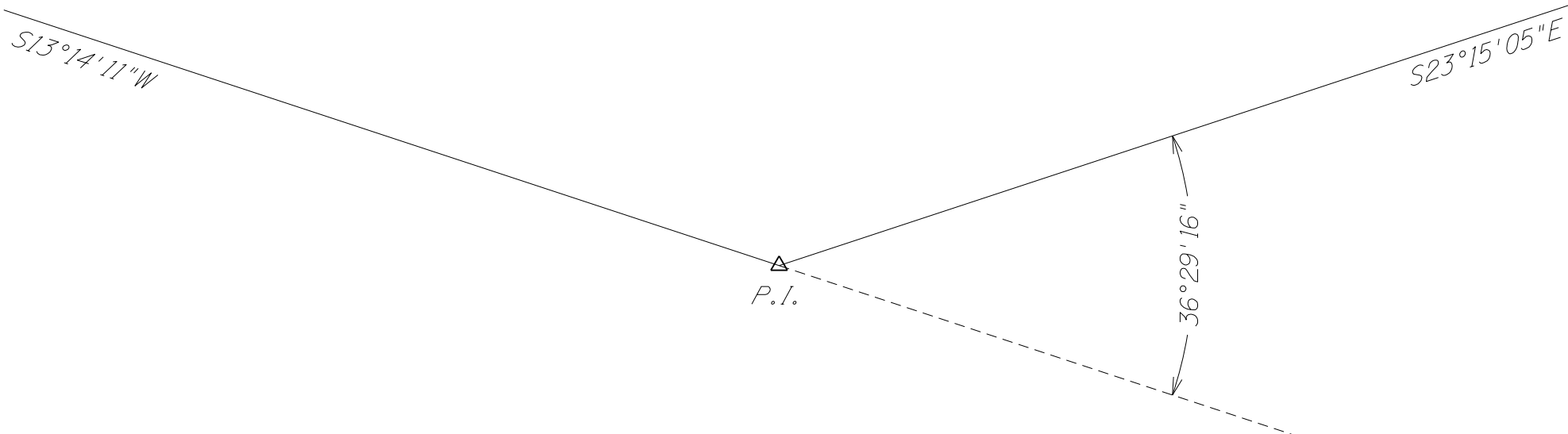


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APPROVED FOR DISTRIBUTION <i>[Signature]</i>	TRANSITION SPIRAL TABLE FOR a=1	PLAN NO. D-57.22

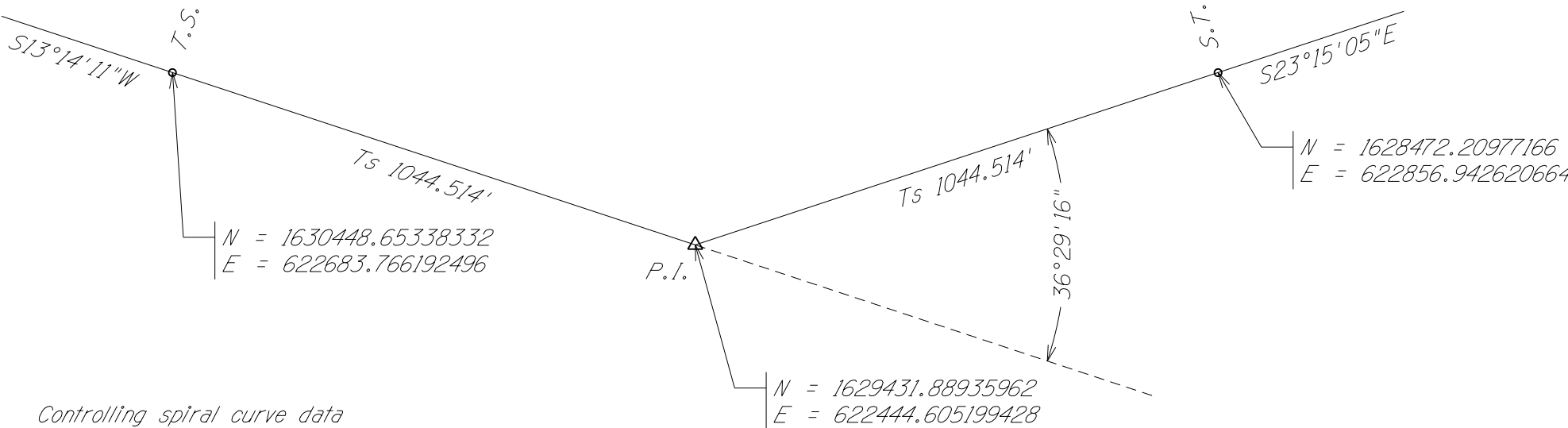
RESULTS OF SURVEY



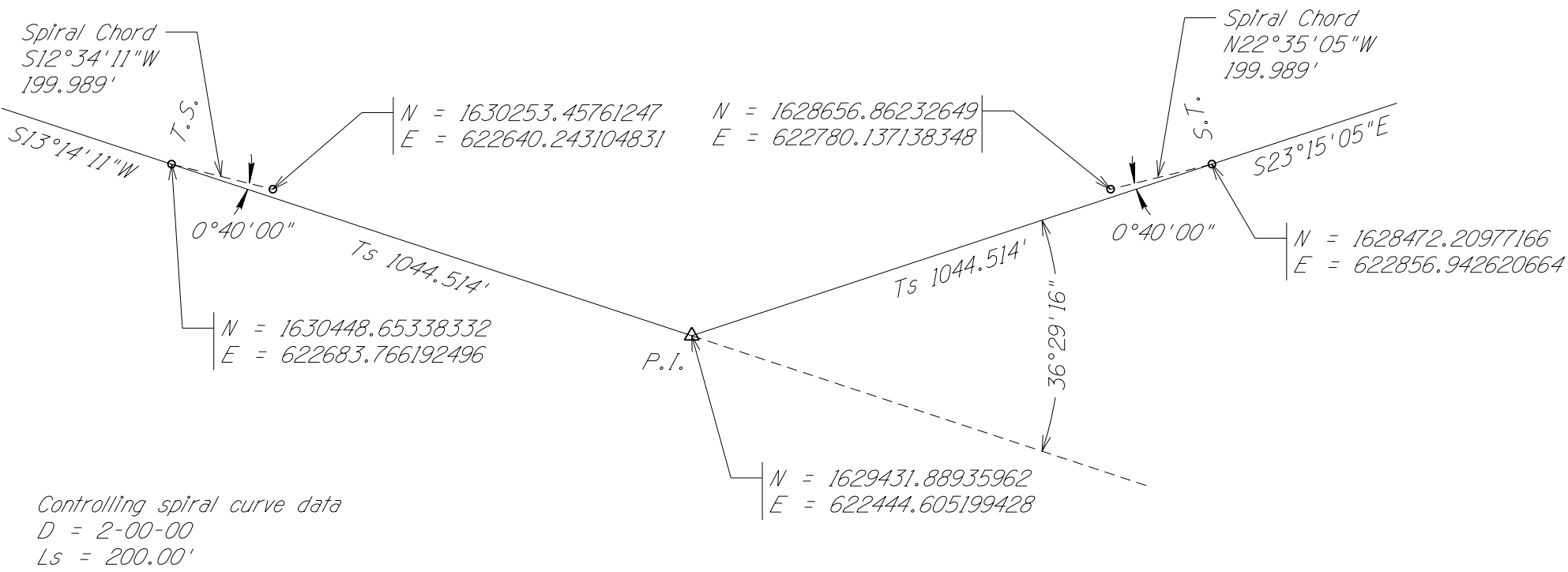
CHANGE ORDER REVISIONS				DRAWING NO.	D-03-T-694	ARIZONA DEPARTMENT OF TRANSPORTATION INTERMODAL TRANSPORTATION DIVISION RIGHT OF WAY PLANS SECTION		PRELIMINARY NOT FOR CONSTRUCTION OR RECORDING --Created-- 7/31/2009 9:50:11 AM
C.O. NO.	DATE	BY	DESCRIPTION OF REVISIONS	SURVEY	Aug 2008	HIGHWAY NAME: WILLIAMS-GRAND CANYON-CAMERON		
				DRAWN/DATE	Jordan/Nov 08	FEDERAL AID NO.: UNASSIGNED		
				ADOT REVIEW	Ken Richmond	PROJECT NO.: 064 CN 185 H7142 OIR		
				 HUBBARD ENGINEERING 480-892-3313 Gilbert, AZ jorume@heaz.us Job # 0812				
				ROUTE NO.:	LOCATION:			
				S.R. 64	Jct. I-40 - Tusayan			
						SHEET RS-17		



Controlling spiral curve data
 $D = 2-00-00$
 $L_s = 200.00'$

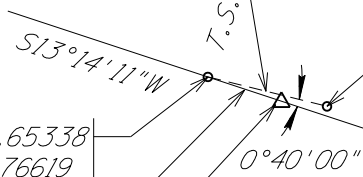


Controlling spiral curve data
 $D = 2-00-00$
 $L_s = 200.00'$



Spiral
 $\Delta = 2^{\circ}00'00''$

Spiral Chord
S12°34'11"W
199.989'



N = 1630448.65338
E = 622683.76619

133.341'

SPI
N = 1630318.8546
E = 622653.2352

Controlling spiral curve data
D = 2-00-00
Ls = 200.00'

N = 1630253.45761
E = 622640.24310

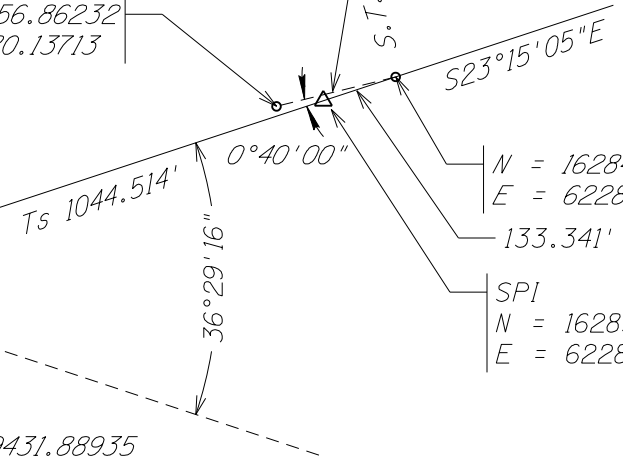
N = 1628656.86232
E = 622780.13713

P.I.

N = 1629431.88935
E = 622444.60519

Spiral
 $\Delta = 2^{\circ}00'00''$

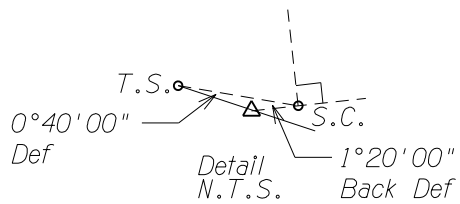
Spiral Chord
N22°35'05"W
199.989'



N = 1628472.20977
E = 622856.94262

133.341'

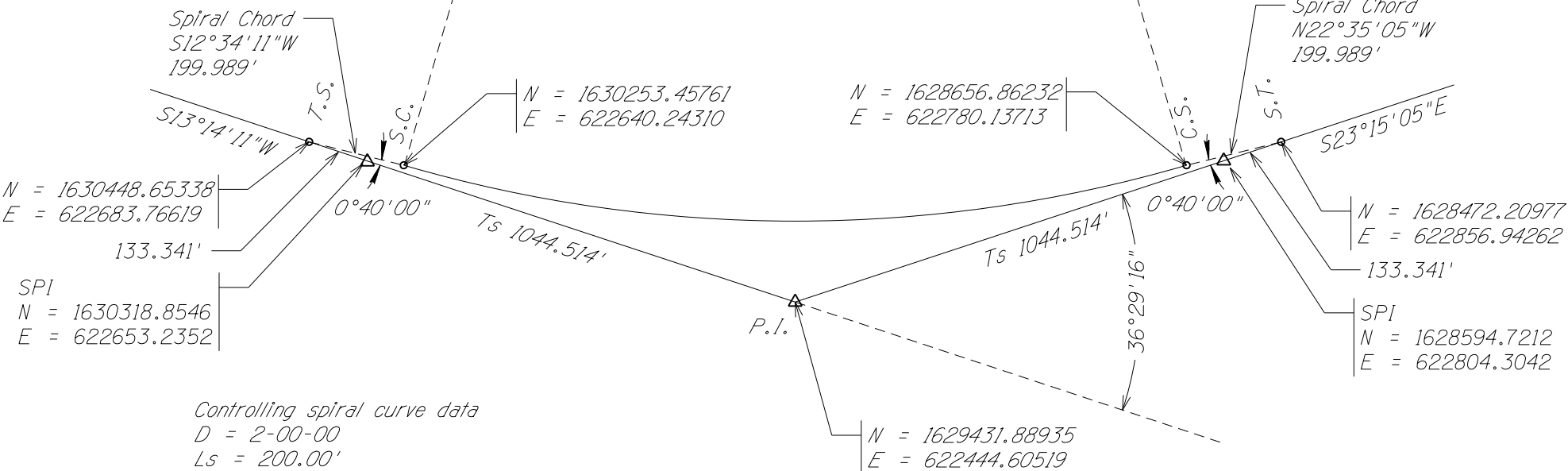
SPI
N = 1628594.7212
E = 622804.3042

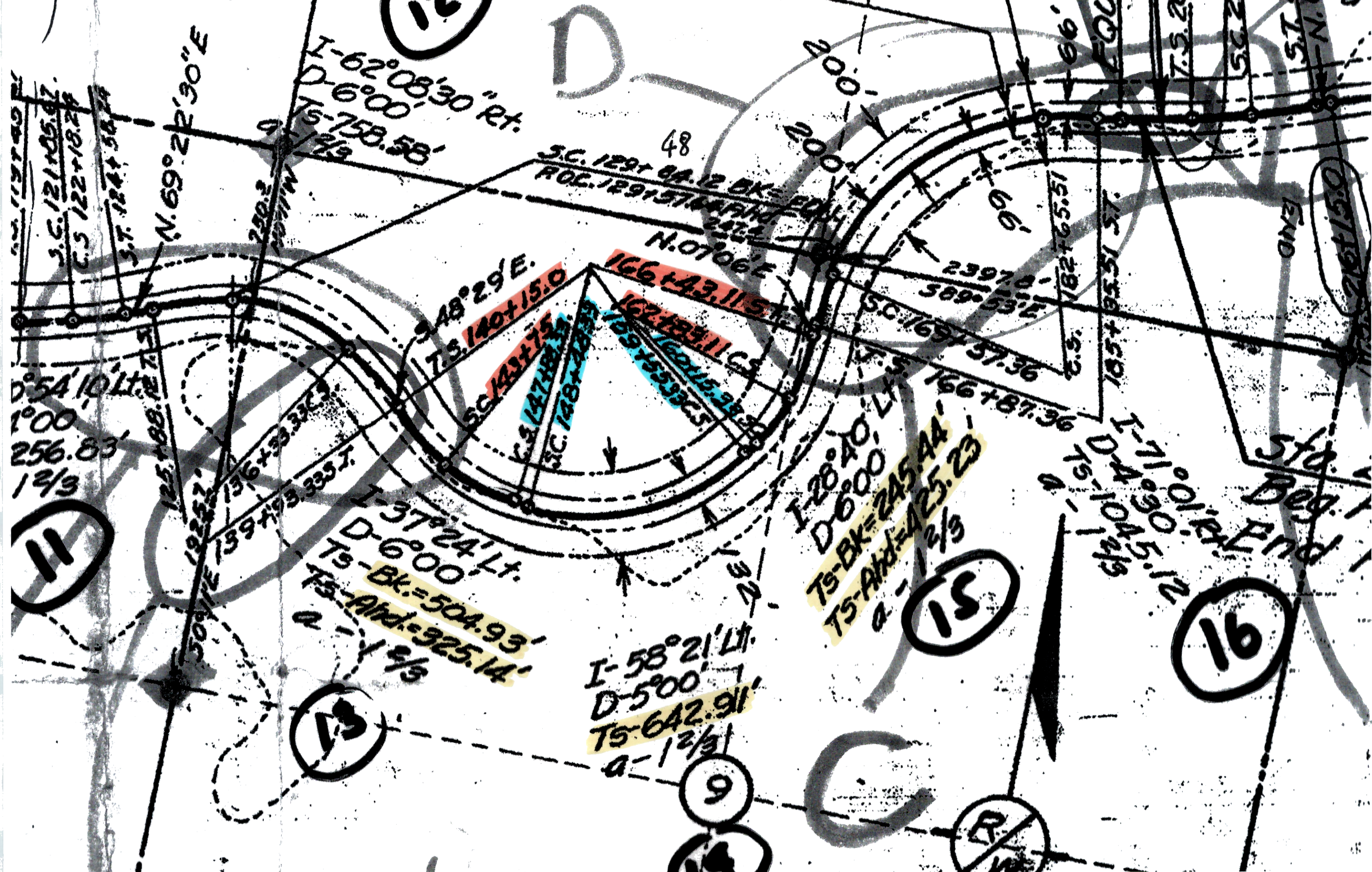


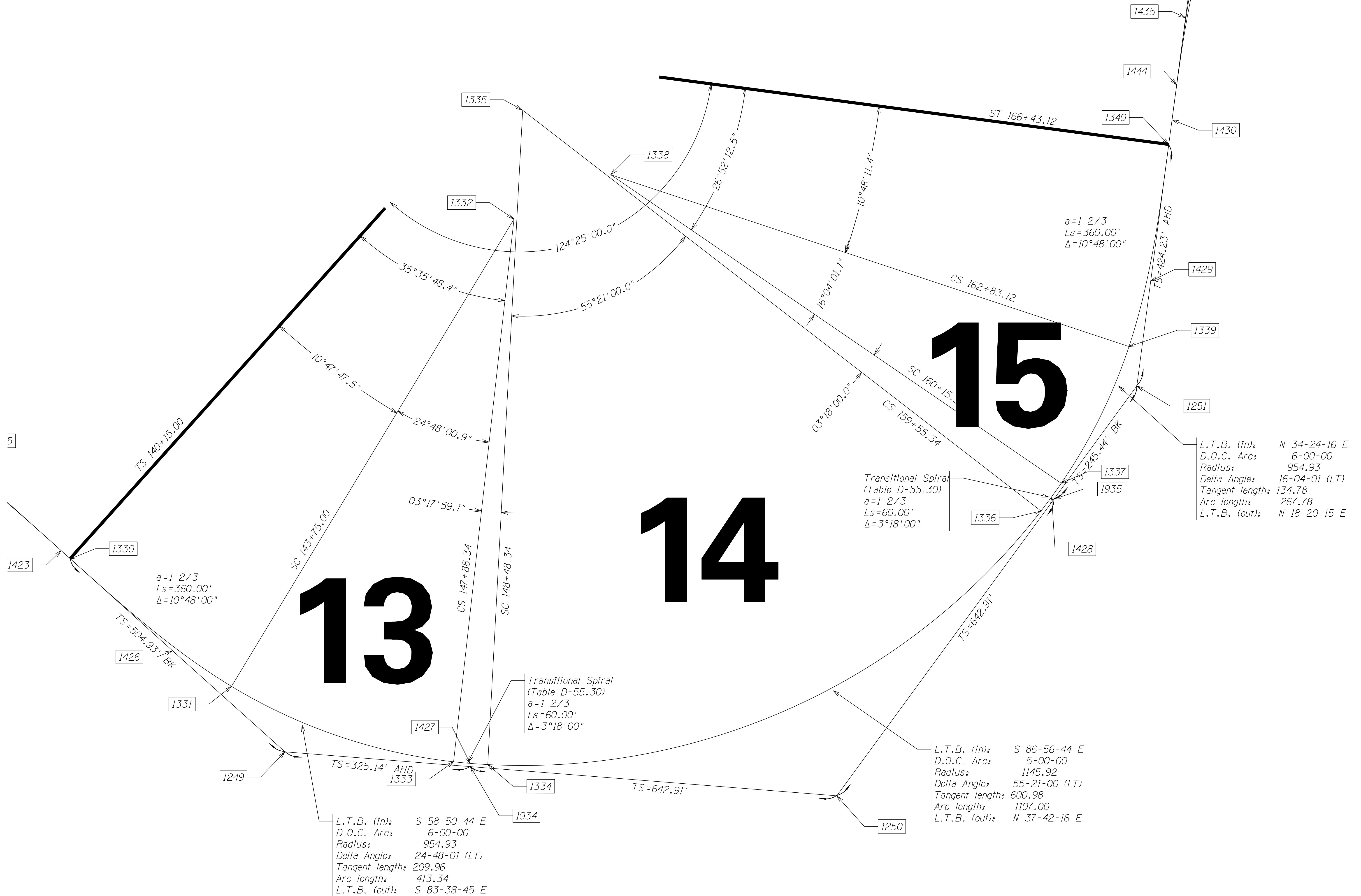
Spiral
 $\Delta = 2^{\circ}00'00''$

Radius Point
N = 1629695.2331
E = 625450.1188

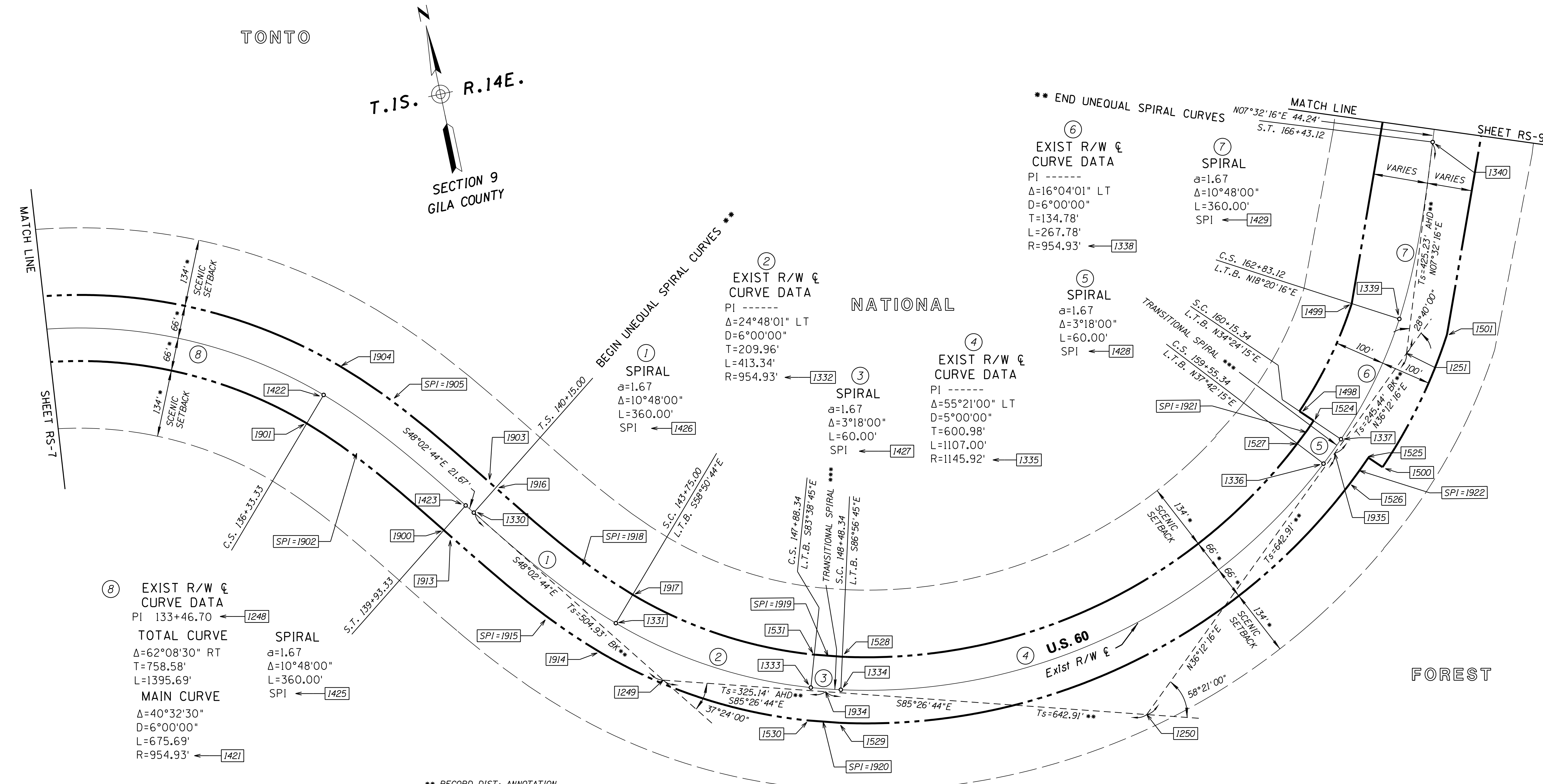
Spiral
 $\Delta = 2^{\circ}00'00''$







RESULTS OF SURVEY




* DENOTES A U.S.F.S. SPECIAL USE PERMIT DATED 6-22-49 WHICH INCLUDES A 132' RIGHT OF WAY WHEREIN THE ROAD WILL BE PARAMOUNT, BUT, NO USE OR OCCUPANCY OTHER THAN FOR CONSTRUCTION AND MAINTENANCE OF THE ROAD IS AUTHORIZED WITHOUT PERMIT FROM THE FOREST SERVICE, AND A 132' WIDE PERMIT TO CONSTRUCT AND MAINTAIN A FENCE, WHERE THE FENCES ARE CONSTRUCTED OUTSIDE OF THE 132' RIGHT OF WAY, NO IMPROVEMENTS OTHER THAN THE FENCES SHALL BE CONSTRUCTED BY THE STATE EXCEPT BY AGREEMENT WITH THE FOREST SUPERVISOR. A 400' WIDE SCENIC STRIP OF "SET BACK" WHEREIN NO CONSTRUCTION OTHER THAN THAT NECESSARY TO THE CONSTRUCTION OF THE HIGHWAY WILL BE ALLOWED, EXCEPT BY SPECIAL PERMIT FROM THE FOREST SERVICE.

** RECORD DIST: ANNOTATION PER STRIP MAP 4-T-137
*** TRANSITIONAL SPIRAL CALCULATED USING FORMULAS AND RELATIONSHIPS SHOWN ON PLAN NO. 55.30, REVISED DATE OF 1/82, OF 1986 ADOT ROADWAY GUIDES FOR USE IN OFFICE AND FIELD.

LEGEND

- 302 Set Monument
- 300 Found Monument
- 301 Calculated position
- Existing R/W monument

CHANGE ORDER REVISIONS				DRAWING NO.	D-04-T-432	ARIZONA DEPARTMENT OF TRANSPORTATION INTERMODAL TRANSPORTATION DIVISION RIGHT OF WAY PLANS SECTION		
C.O. NO.	DATE	BY	DESCRIPTION OF REVISIONS	SURVEY	April 2006	HIGHWAY NAME:	PHOENIX - GLOBE	PRELIMINARY NOT FOR CONSTRUCTION OR RECORDING --Created-- 8/14/2009 8:19:50 AM
				DRAWN/DATE	J Crume/2006	FEDERAL AID NO.:		
				ADOT REVIEW	C. Woodford	PROJECT NO.:	060 GI 236 H6140 01R	
				 HUBBARD ENGINEERING 480-892-3313 Gilbert, AZ jcrume@haz.us Job # 26015				
				ROUTE NO.:	U.S. 60	LOCATION:	County Line - Pinto Valley	SHEET RS-8

0 50 100
Scale
060rw236-rs08.dgn